From Martin Lof's type theory to Homotopy type theory

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Martin Lof's type theory (MLTT) is formalism for doing constructive mathematics. Its computational nature and expressive power provides a basis for many programming languages and proof assistants. It supports proof relevant mathematics, where proofs are first class objects that may appear in further propositions (also called dependent types). Basic type to allow this is equality on two objects (proofs) of a given type.

It was known in the 90's that if types are considered as categories then equality type between two elements of a type may be thought of type of morphisms between these objects of the category. The iterated equality type construction associates higher groupoid structure to a type.

In traditional constructive mathematics this higher groupoid structure is collapsed to the first level by extensionality axiom. In the last decade it was realized by Voevodsky and others that if types are used as spaces and equality as paths between points of a space then higher groupoid structure of types corresponds to the higher homotopy groupoid of a space.

This has led to extension of MLTT by univalence axiom and higher inductive types, called homotopy type theory (HoTT). HoTT may be used to reason about homotopical structure of spaces formalized in synthetic way. It also offers mechanization and use of proof assistants in this reasoning.

In this lecture series, I plan to give a self contained introduction to MLTT and show some links with HoTT.